Highly sensitive label-free coupled resonator Fabry–Perot self-referencing photonic biosensor

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There is a demand for the development of portable easy-to-use on-chip biosensors capable of single molecule detection. In this paper, we propose a new ultra-sensitive label-free optical refractive index sensor consisting of a Fabry–Perot resonator evanescently coupled to a microring resonator. The coupling between the Fabry–Perot standing wave mode and degenerate whispering gallery modes of the microring leads to a splitting of the resonances proportional to the coupling. Choosing the coupling region to be a microfluidic channel through which the analyte solution is pumped leads to a change in the splitting proportional to the index of refraction of the analyte that is immune to common-mode noise sources in the resonators such as thermal noise. The high sensitivity of the coupling to the analyte index combined with the reduced noise leads to significant improvement in the detection limit down to $\sim 10^{-9}$ RIU. © 2017 Optical Society of America

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1. INTRODUCTION

In recent years, there has been a strong push toward the development of ultrasensitive, portable, and easy-to-use on-chip biosensors capable of operating in the field in areas such as healthcare, environmental monitoring, and homeland security where standard methods are precluded due to cost, environment, or lack of skilled personnel. Label-free optical refractive index (RI) sensors have gained particular attention because of the capability of ultrasensitive detection of index changes \cite{1,2}. Thanks to the polarizability of each molecule when interacting with light combined with the relative ease of measuring very small changes in the optical index of refraction by an interferometer or the resonance shift of an optical resonator, RI sensors can detect changes in the surrounding index of refraction as small as $10^{-8}$ RIU (refractive index units) corresponding to detection of bulk concentrations as small as $\sim$ picograms/milliliter (pg/mL) \cite{3}. Since the molecules are neither labeled nor altered, this technique allows for simple, fast, and relatively inexpensive detection compared to optical fluorescence techniques.

Currently, there exist several types of RI sensors, but the most common of which are surface plasmon resonances (SPRs) \cite{3,4}, Bragg gratings, Mach–Zehnder interferometers \cite{1}, and optical microresonators. SPR sensors have been shown to offer sensitivities in the nanomolar range but are relatively large in size and require complex integration. Bragg gratings \cite{5} and Mach–Zehnder interferometers \cite{6}, on the other hand, can be easily miniaturized and offer high on-chip integration but at the cost of lower sensitivity. Integrated optical microresonator RI sensors have radii of $\sim 10–100 \ \mu m$, making them among the smallest RI sensors, which permits large-scale integration and multiplexed operation on a chip \cite{7}. The mechanism for RI sensing is the dependence of the resonator modes’ resonance frequency on the effective index of refraction. The effective index depends not only on the RI of the resonator material but is also affected by the index of refraction of the local environment surrounding the resonator due to the resonator mode’s evanescent field. The shift in the resonance frequency due to a perturbation of the environment RI can be measured directly from the transmission spectrum using a tunable laser along with a spectrometer.

However, the resonance frequency of a microresonator is also affected by noise sources such as temperature fluctuations, which is due to the thermo-optic effect change in the RI. This type of noise reduces the detection limit of the microresonator. There have been a few microresonator designs proposed that seek to cancel the effect of noise by using two degenerate resonator modes subject to the same noise sources \cite{8–10}. Specifically, vertically stacked microring resonators interacting with a microfluidic ring channel \cite{8} and a self-referencing optofluidic ring resonator sensor have been tested for added noise suppression through common noise cancellation \cite{10}.

2. MODEL

Here we propose a new type of microresonator RI sensor that utilizes a microring (MR) resonator of radius $R$ coupled to a Fabry–Perot (FP) resonator of length $L$, as illustrated in
component of the FP standing wave mode is excited, of phase matching, only the left-propagating traveling wave are assumed to have very high reflectivities. is also evanescently coupled to a FP whose ends at

degenerate counterpropagating whispering gallery modes of the microring are coupled evanescently to a single standing wave mode of the FP, also assumed to be degenerate with the microring modes. The three coupled modes exhibit eigenfrequencies that are split symmetrically about their common resonance frequency by an amount equal to the evanescent coupling between the resonators. The strong dependence of the evanescent coupling on the index contrast between the resonator and the surrounding environment leads to a mode splitting that is very sensitive to small perturbations in the RI of the environment. By choosing the coupling region to be a microfluidic channel between the MR and FP through which the analyte solution is pumped leads to a high sensitivity to the analyte RI in a very small sensing volume compared to other microresonator sensors. Moreover, since the frequency splitting of the coupled modes is independent of the original modes’ resonance frequency, noise sources that are common to all of the resonator modes, including temperature fluctuations, are canceled when measuring the splitting of the eigenfrequencies, leading to an improved detection limit.

Light from a probe laser is evanescently coupled from the waveguide into the MR resonator of radius $R$, as shown in Fig. 1. Specifically, the probe field amplitude from the waveguide, $S_{in} \exp(\text{i} \omega t)$, is coupled at the rate $k_{MW}$ to the counterclockwise whispering gallery mode with amplitude $b_2$. The MR is also evanescently coupled to a FP whose ends at $y = 0$ and $L$ are assumed to have very high reflectivities $r_1 = r_2 \approx 1$. The mode $b_2$ is coupled to the FP mode at the rate $k_{MF}$ but because of phase matching, only the left-propagating traveling wave component of the FP standing wave mode is excited, $a_{-k} \exp[-iky]$ as determined by the reflectivities, $a_{-k} = -r_1 a_{+k}$. Similarly, at the other end, the boundary condition is $a_{+k} \exp[-ikL] = -r_2 a_{-k} \exp[ikL]$. Again, due to phase matching, the right traveling wave $a_{+k}$ is coupled only to the clockwise-propagating whispering gallery mode of the MR, $b_1$, at the same rate $k_{MF}$. Since the FP eigenmodes are standing waves with $kL = 2\pi m_f$, where $m_f$ is the longitudinal mode number, the boundary conditions then reduce to $a_{+k} = -a_{-k} = a_0$, where $a_0$ is the amplitude of the standing wave. The standing wave field is then proportional to $(a_{-k} \exp[iky] + a_{+k} \exp[-iky])/2i = a_0 \sin(ky)$, which has nodes at $y = 0$ and $L$. As a result, we write the coupled mode equations for the evolution of the FP standing wave mode and MR modes as

$$\frac{da_0}{dt} = (\delta \omega_{FP} + \gamma_{FP}/2)a_0 - ik_{MF}(b_1 - b_2), \quad (1)$$

$$\frac{db_1}{dt} = (\delta \omega_{MR} + \gamma_{MR}/2)b_1 - ik_{MF}a_0, \quad (2)$$

$$\frac{db_2}{dt} = (\delta \omega_{MR} + \gamma_{MR}/2)b_2 - ik_{MF}a_0 - ik_{MW}S_{in}^{\text{out}}. \quad (3)$$

Here $\omega_{FP}$ is the FP mode frequency and $\gamma_{FP}$ is the decay rate of the mode energy, while $\omega_{MR}$ is the resonance frequency of the MR modes and $\gamma_{MR}$ is the decay rate of their energy. Here we have included in both resonators a fluctuating component to the resonance frequency $\delta \omega_{MR/FP} = (\delta \omega_{MR/FP}/n_{MR/FP})\omega_{MR/FP}$, where $n_{MR/FP}$ is the effective index of the two resonators and $\delta \omega_{MR/FP}$ is the shift of the effective index as a result of temperature fluctuations or other noise sources such as mechanical strain. The decay rates can be further expressed as $\gamma_{FP} = \alpha \omega_{FP}$ and $\gamma_{MR} = \alpha \omega_{MR} + |K_{MW}^2| v_{MR}/2\pi R$, where $v_{MR}$ and $v_{FP}$ are the group velocities of the MR and FP modes, respectively, and $\alpha$ is loss per unit length in the resonators [11,12]. $|K_{MW}^2|$ is the ratio of the power coupled into the MR to the incident power from the waveguide such that $0 \leq |K_{MW}^2| \leq 1$ and $k_{MW} = |K_{MW}^2|v_{MR}/2\pi R$. Moreover, the outgoing transmitted wave in the waveguide includes the signal that is coupled out of the MR into the waveguide as

$$S_{out} = S_{in}^{\text{out}} - ik_{MW}b_2. \quad (4)$$

As per Ref. [11], the modulus square of the intraresonator fields $|a_0|^2$, $|b_1|^2$, and $|b_2|^2$ have units of energy, while the fields in the waveguide $|S_{in}|^2$ and $|S_{out}|^2$ are in units of power. It is important to note that the value of $k_{MW}$ and equivalently $|K_{MW}^2|$ have little effect on the overall sensitivity of the system since the resonance frequencies of the device are independent of $|K_{MW}^2|$. $|K_{MW}^2|$ does affect the linewidth of the resonances provided $|K_{MW}^2| > 2\pi R\alpha$, which is true usually even for low-loss waveguides. However, the value of $|K_{MW}^2|$ is proportional to the gap between the microresonator and the waveguide, which can be tailored during the fabrication process. Moreover, the coupling can be further precisely tuned by changing the index of refraction in the coupling region through the use of the thermo-optic effect and heaters applied above the coupling, as described further in Section 3 [13].
Fig. 2. Transmission spectrum calculated from coupled mode equations (dashed line) and transfer matrices (solid line) for $K_{MR}^2 = K_{MW}^2 = 0.001$, $R = 50 \mu m$, and $\alpha = 0 \text{ dB/cm}$. Mode numbers used in the transfer matrices are $m_{fp} = 10$ and $m_{mr} = 20$ for the FP and MR resonators, respectively. The length of the FP was chosen such that $\omega_{fp} = \omega_{MR} = \omega_0$ and a group velocity for both resonators of $v_{fp} = v_{MR} = 2.2 \times 10^5 \text{m/s}$. The inset shows the resonator coupling $K_{MR}^2$ versus the location of the rightmost resonance comparing the transfer matrix solution (solid line) to the coupled mode solution (dashed line).

As a verification of the coupled mode equations, we compared the steady-state solutions of Eqs. (1)–(3) to a transfer matrix model given in Appendix A for $\omega_{FP} = \omega_{MR} = \omega_0$ and $\delta \omega_{FP/MR} = 0$. The transmission of the FPMR sensor, which is given by $T = |S_{out}/S_{in}|^2$, is shown in Fig. 2, where one can see three distinct resonances arranged symmetrically around $\omega_0$. An assessment of the accuracy of the coupled mode equations in comparison to the transfer matrices is shown in the inset, where the frequency of the rightmost resonance is plotted as a function of $K_{MF}^2 = \kappa_{MF}^2(2\pi R/v_{MR})^{1/2}(2L/v_{FP})^{1/2}$, which is the normalized power coupling between the resonators. In the weak coupling regime $K_{MF}^2 \ll 1$, there is strong agreement between the two models, which, however, begin to diverge for large coupling $K_{MF}^2 \to 1$ as to be expected.

The locations of the three resonances seen in Fig. 2 can be easily determined from the eigenvalues of the homogeneous part of Eqs. (1)–(3) for $\gamma_{FP} = \gamma_{MR} = 0$:

$$\omega_0 = \Omega_{MR}, \quad \omega_\pm = \frac{1}{2} \left( \Omega_{MR} + \Omega_{FP} \pm \sqrt{\delta \kappa_{MF}^2 + (\Omega_{MR} - \Omega_{FP})^2} \right). \quad (6)$$

where $\Omega_{FP} = \omega_{fp}$, $\Omega_{MR} = \omega_{MR}$, $\delta \kappa_{MF} = \kappa_{MF} - \kappa_{MF}/\pi$, and $\delta \omega_{fp}$ and $\delta \omega_{MR}$. By choosing $\Omega_{FP} = \Omega_{MR}$, the splitting of the modes $d_\pm$ is directly proportional to the coupling $\kappa_{MF}$ and independent of the noise fluctuations $\delta \omega_{fp}$ and $\delta \omega_{MR}$. This can be accomplished by designing the resonators to have not only equal resonance frequencies $\omega_{MR} = \omega_{FP}$ but also equal effective indices for the modes $n_{MR} = n_{FP}$. Equal effective indices along with the assumption that any variations of the temperature are the same in the MR and FP, which should be readily satisfied since the overall device dimensions would be less than 100 \mu m, causes $\delta \omega_{FP} = \delta \omega_{MR}$. Under these conditions, the resonance splitting is simply $\Delta \omega = \omega_+ - \omega_- = 2\sqrt{\kappa_{MF}}$.

The eigenmode with frequency $\omega_0$ corresponds to a superposition of the MR whispering gallery modes, $d_0 = (b_1 - b_2)/\sqrt{2}$. Interestingly, this state is mathematically equivalent to the dark state observed in a coherently driven three-level $\Lambda$ atom shown in Fig. 1 that has found many applications in spectroscopy, including stimulated Raman adiabatic passage [14] and electromagnetically induced transparency [15–17]. This follows from the mathematical equivalence of our coupled mode equations to the Schrödinger equation for a $\Lambda$-atom with a pair of atomic ground states coupled to a single excited state by two resonant lasers [18,19]. The whispering gallery MR modes play the role of the two ground states $\{|g_\pm\rangle\}$, while the FP mode is represented by the atomic excited state $|e\rangle$ with the resonator coupling $K_{MF}$ playing the role of both resonant lasers acting on the atomic states. In our case, the dark state $d_0$ is formed by the interference between the MR modes mediated by the FP mode. However, for the purpose of index sensing, it is the other eigenstates with eigenvalues $\omega_\pm$ that are important. These two states are a superposition of the three modes, $d_\pm = (b_1 \pm \sqrt{2} \alpha_0 + b_2)/2$.

The analyte to be detected flows through a fluidic channel in the gap between resonators. The RI in the gap $n_\ell$ is changed by the presence of the analyte, which subsequently perturbs $K_{MF}$, changing the frequency separation $\Delta \omega$. The transmission as a function of the probe frequency $\omega$ for different changes of the index $\Delta n_\ell$ is shown in Fig. 3, where one can see the linear shift of the side resonances while the central resonance remains fixed. One must keep in mind that the probe transmission is measuring the eigenfrequencies of Eqs. (5) and (6) of the coupled resonator system and not the resonance frequencies of the individual resonator modes—all of which are degenerate. In the limit of vanishing $K_{MF}$, there would only be single transmission resonance at $\omega_0$, representing these degenerate modes. For finite $K_{MF}$, however, these degenerate modes no longer represent the eigenstates of the system. Instead, the mixing of the resonator modes leads to new eigenmodes, which represent different hybridizations of the original modes, $d_0$ and $d_\pm$, having different spatial patterns in the resonators and hence frequencies. As a result, the frequencies of the eigenmodes will no longer be the
same as the original modes but will now be split with one moving toward higher frequency and the other toward lower frequency as the coupling strength increases. This behavior is equivalent to the dressed states formed in atoms when two or more energy levels are coupled by a resonant laser and is indeed generic to any system in which a weak coupling is introduced between energy or frequency modes of a physical system.

The calculations here are not specific to either a bulk change of the RI of the fluid in the channel or due to a change of the index at the surface of the resonators due to binding of the analyte to a functionalized resonator surface in the channel as this would depend on the analyte being detected. Instead, the formulas presented here simply relate the changes in the coupling coefficient to the change in index in the gap between the resonators. This index change $\Delta n_z$ could either be expressed in terms of the bulk index change of the channel fluid or the change in the index of refraction due to surface chemistry as a result of functionalization of the channel. Regardless, the sensitivity to the index change is defined as

$$S = \frac{d\Delta \omega}{dn_z},$$

(7)

which under the assumptions of the preceding paragraph reduces to $S = 2\sqrt{2}dK_MF/dn_z$.

To derive an expression for the coupling $K_MF$ under the assumption that the waveguide segments of the two resonators can be treated as 1D slabs. The coupling strength per unit length when the distance between the waveguide centers of the two resonators is 2 along the direction $x$ is determined by the overlap integral

$$\xi(x) = \epsilon_0\omega \int_0^\infty (n_1^2 - n_e^2) \epsilon_{MR}(x) \epsilon_{FP}(x) dx/4,$$

where $\epsilon_{MR}(x)$ and $\epsilon_{FP}(x)$ are the normalized modal fields in the individual resonators, $\epsilon_0$ is the permittivity of free space, and $n_1$ is the RI for the resonator material. The coordinate $x$ is the position around the perimeter of the MR measured from the point of minimum separation between the resonators.

Due to the MR curvature, the interaction length $L_{int}$ over which power is exchanged is small compared to the lengths of both resonators, $L_{int} \ll R, L$, and the coupled amplitudes do not change appreciably over the length so that we approximate $\xi(x) = \xi(0) = \xi$.

For constant $\xi$ the overlap integral $\xi$ over the length $L_{int}$ [20], $K^2_{MF}$ can be calculated using the analytical expression for a straight directional coupler as [21]

$$K^2_{MF} = \frac{2\xi^2}{\xi^2 + (\delta \beta/2)^2} \sin^2\left(\sqrt{\xi^2 + (\delta \beta/2)^2}L_{int}\right),$$

where $\delta \beta$ is the phase mismatch between the resonator modes that we assume here to be 0. Following the treatment of $\xi$ in Ref. [11], the change in $K^2_{MF}$ due to the RI change caused by the analyte $\Delta n_z$ is

$$\Delta K^2_{MF} \approx \frac{4n_0}{NA^2} \Delta n_z,$$

(8)

where the numerical aperture (NA) is $NA^2 = n_1^2 - n_0^2$, where $n_0$ is the RI of the microfluidic channel without the analyte, $n_e = n_0 + \Delta n_z$.

### 3. RESULTS

Unless stated otherwise, for all calculations including the figures, we assumed that the resonators were made out of CYTOP [22] with Teflon AF [23] cladding, except in the microfluidic channel, which have indices of refraction of $n_{CYTOP} = n_1 = 1.3335$ and $n_{TeflonAF} = 1.2976$ at a wavelength of 1.55 μm. The losses within the resonators, corresponding to CYTOP, were taken to be $\alpha = 0.12$ dB/cm while for the group velocities $v_{MR} = v_{FP} = 2.2 \times 10^4$ m/s [24]. The width of the microfluidic channel was assumed to be 680 nm and the interaction length of $L_{int} = 1900$ nm. Similarly, this could be accomplished with a number of other materials such as resonators made out of PMMA with a CYTOP cladding [25], in order to avoid any possible degradation in the quality factor of the resonator when exposed to water. Additionally, PMMA’s index of refraction can be adjusted from 1.5 to 1.33 by simply adjusting its fluorination levels [26]. It is also important to note that the cladding layer was chosen to be sufficiently thick outside the coupling region so that the evanescent field does not have the same response in the FPMR as it does in the AP.

To evaluate the sensitivity of the FPMR system, we compare it to a single microring resonator all-pass (AP) system in Fig. 4, which can be described by the single coupled mode equation:

$$\frac{db}{dt} = \left(i[\omega_{MR} + \delta \omega_{MR} + \Delta \omega_{s}] - \gamma_{MR}/2\right)b_2 - iK_MW s_{in}^e,$$

(9)

along with the same Eq. (4) for the transmitted field. The AP is critically coupled, $K^2_{MW} = 2\pi Ra$, and is assumed to have the same geometric area as the FPMR since the FP is smaller than the diameter of the MR and both the AP and MR have the same radius. It is important to note that in the FPMR system, the sensing occurs in the coupling region between the FP and MR, while in the AP setup, the entire circumference of the resonator is exposed to the solution containing the analyte. For the AP resonator, the effective index of refraction is perturbed by the presence of the analyte interacting with the evanescent field. To first order in the changes of the RIs, the change in the effective index is $\Delta n_{AP} = \eta \Delta n_z + (1 - \eta) \Delta n_r$, where $\eta$ is the fraction of the total power of the resonator mode in the evanescent field. This results in the resonance shift $\Delta \omega_{AP} = \eta \omega_{MR} (\Delta n_z / n_{MR})$ due to the analyte. The sensitivity of the AP is simply the shift of the single transmission resonance,

$$S = \Delta \omega_{AP}/n_{MR},$$

Figure 4 shows the splittings between the resonances as a function of $\Delta n_z$ from which one can determine the sensitivity. It is evident that when $NA^2 = 0.01$, the coupled system has an order of magnitude greater sensitivity than the AP for equal $\Delta n_z$. This is despite the fact that the sensing area of the AP is much larger. The AP is though more stable to changes in the numerical aperture. For instance, a shift in $NA^2$ from 0.01 to 0.1 resulted in a decrease in the sensitivity by less than 14% of the AP, while for the coupled resonators, the reduction was approximately a factor of 10. This follows directly from Eq. (8) and is due to the strong dependence on NA of the penetration depth of the evanescent fields of the resonators into the microfluidic channel, whose mutual overlap determines the
coupling $K_{MF}$. By contrast, NA affects $\eta$ for the AP more weakly. Despite this, the FPMR still has a slightly larger shift in the resonance than the AP for $\text{NA}^2 = 0.1$.

However, the greatest advantage of the coupled resonator system is the common-mode noise cancellation leading to better detection limits than the AP setup without the need of any external noise-suppression techniques. From Eq. (9), the absolute position of the resonance is at $\omega = \omega_{\text{MR}} + \Delta \omega_n + \delta \omega_{\text{MR}}$, which will randomly fluctuate due to the noise term $\delta \omega_{\text{MR}}$, thereby masking the desired signal given by $\Delta \omega_n$. By contrast, for the FPMR, any reciprocal noise in the resonators affecting the index of refraction can be made to cancel out when measuring the resonance splitting of the $d_{\pm}$ modes. The detection limit (DL), which expresses the minimum RI change the sensor can reliably measure, is determined by the uncertainty in determining the location of the resonance linecenters. For the AP, the detection limit is determined by $\delta \omega_{\text{MR}}$ as well as laser noise and the resolution of the spectrometer being used. By contrast for the FPMR, only the latter two sources of uncertainty affect the DL. Under the assumption that the resolution of the spectrometer is perfect, the detection limit of the FPMR can be expressed using the method described in Ref. [27] as

$$\text{DL} = \frac{\Delta \lambda}{S_2 \cdot 4.5 \cdot \text{SNR}^{\frac{3}{2}}}, \quad (10)$$

where SNR is the signal-to-noise ratio of the laser, chosen to be $\text{SNR} = 80$ dB, which is equivalent to a shot-noise limited system with an input power $P_{\text{in}} = 1 \text{ mW}$. Additionally, $S_2$ is the sensitivity in wavelength units (nm/RIU) and $\Delta \lambda$ is the full width at half-maximum of the resonances. The detection limit for the coupled resonator system is shown in Fig. 5 with $\text{NA}^2 = 0.01$, $R = 50 \mu$m, $\alpha = 0.12 \text{ dB/cm}$, $K_{MF}^2 = 2\pi R a$, and the mode numbers $m_{FP} = 10$ and $m_{MR} = 20$ for the FP and MR, respectively. For $K_{MF}^2 = 0.75$, this yields a detection limit of $\text{DL} \approx 1.9 \times 10^{-9}$ RIU. Even if one were to ignore the resonance fluctuations $\delta \omega_{\text{MR}}$, the detection limit of the AP would still only be $1.7 \times 10^{-7}$ RIU due to the AP’s reduced sensitivity and larger linewidth. However, if we were to take into account thermal noise in the AP index sensor as laid out in Ref. [27], leading to an uncertainty in the linecenter measurement of $10 \text{ fm}$, there is a further reduction of the detection limit to $9.6 \times 10^{-6}$ RIU still assuming a SNR = 80 dB. The FPMR consequently has a 2–3 orders of magnitude better detection limit. A further evaluation of the FPMR detection limit is displayed in Fig. 5, where one sees that for increasing $K_{MF}$ the detection limit improves. This is because the increase of $K_{MF}$ is due to a decrease of NA for fixed waveguide separation, which increases the sensitivity. Lower resonator losses also lead to a better DL in Fig. 5 because of the smaller $\Delta \lambda$, as one would expect.

Finally, we address how the misalignment of the resonance frequencies of the Fabry–Perot and microring due to either fluctuations in the effective indices such as caused by temperature fluctuations or fabrication errors in the resonator dimensions negatively affect the detection limit as well as how to mitigate these effects. First, the effective indices of the resonator modes are affected by the temperature due to the thermo-optic effect. Namely, the change in the index of refraction of a material due to a temperature change $\Delta T$ is $\Delta n = \frac{\partial n/\partial T}{\text{PMMA}} \Delta T$, where $\partial n/\partial T$ is the thermo-optic coefficient. In order for our biosensor to be independent of temperature fluctuations, the temperature-induced fluctuations of the resonance frequencies must be much smaller than the frequency splitting being measured. Specifically, if the following condition is satisfied:

$$\frac{2}{\pi} \frac{\Delta K_{MF}^2}{m_{FP} m_{MR}} \gg \left( \frac{\delta n_{MR}}{n_{MR}} - \frac{\delta n_{FP}}{n_{FP}} \right)^2,$$

then the detection limit of the sensor is unaffected by the temperature fluctuations. For instance, if we use the thermo-optic coefficients of CYTOP and PMMA, which are $\partial n/\partial T_{\text{CYTOP}} = -9.7 \times 10^{-3} \text{ K}^{-1}$ and $\partial n/\partial T_{\text{PMMA}} = -1 \times 10^{-4} \text{ K}^{-1}$ [28,29], respectively, then the temperature fluctuations would have to be less than approximately one micro-Kelvin. Such a small temperature differential should be
achievable over the device dimensions of \(\sim 100 \mu m\). However, it is also possible to design athermal waveguides for the resonators that have significantly reduced sensitivity to temperature changes so that the resulting change in the effective indices of the resonator modes due to \(\Delta T\) is much smaller than that of the RI change of the materials out of which the resonator waveguides are fabricated. This is accomplished by using core and cladding materials for the waveguides that have thermo-optic coefficients of similar magnitude but opposite signs \([30,31]\). Since the effective index depends on the RI of both the core and cladding materials, an increase of the RI of the core due to a temperature change is cancelled out by a corresponding decrease of the RI of the cladding, thereby rendering the effect of temperature fluctuations negligible whatever the source. One such possible material combination would be if resonator waveguides were made out of PMMA with a silica (SiO\(_2\)) cladding since the two materials have thermo-optic coefficients of opposite signs. Another combination would be a silica waveguide core with a CYTOP cladding since they have opposite thermo-optic coefficients of the same magnitude \([28,29]\).

Similarly, a mismatch of the resonance frequencies could occur due to fabrication errors. A typical tolerance of an electron beam lithography system such as the JEOL 6300FS \([32]\) is several nanometers in terms of positional accuracy within the field \(\leq \pm 9 \text{ nm}\), where it is safely assumed that the percentage outside the confidence interval is 0.001% or 4.4\(\sigma\) and as a result the length uncertainty of the resonators can be \(\sigma_p \approx 2 \text{ nm}\). This would represent a relatively large mismatch of the FP and MR resonance frequencies reducing the detection limit by roughly 1–2 orders of magnitude. However, unlike temperature fluctuations, which are a random process, the fabrication errors are fixed and can be compensated for postfabrication by two methods, either done separately or together for even greater matching of the resonances. The first would be to adjust the length of the FP postfabrication in order to match its resonance to the MR. This can be achieved using a focused ion beam in order to polish down the Fabry–Perot ends before the reflective ends are added. The second method would be to use thermo-optic heaters to match the two resonances. Specifically, chromium resistors acting as electric heaters can be deposited on top of the resonators’ waveguides in order to finely tune each of their resonance frequencies by the thermo-optic effect \([13]\). This latter method though supposes that the waveguides are not athermal as discussed above.

4. CONCLUSION

In conclusion, we have presented a new highly sensitive self-referencing label-free refractive index sensor that senses the change in the index of refraction in a microfluidic channel in the evanescent coupling region between a microring and Fabry–Perot resonator. More importantly, the eigenmodes of the coupled resonator system have resonance frequencies that are independent of reciprocal noise sources such as temperature fluctuations provided the resonators are designed to have equal mode frequencies and effective indices of refraction. This noise cancellation in the FPMR resonance frequencies means that the detection limit is limited only by the probe laser’s signal-to-noise ratio and the resolution of the spectrometer used. By comparison to a traditional all-pass microring resonator index sensor, it was shown that the detection limit of the coupled resonator system was more than 100 times better than the AP for equivalent device parameters. Additionally, it should be feasible to fabricate the FPMR in a clean room with electron beam lithography using currently available technologies such as CYTOP as mentioned above.

APPENDIX A

Here we describe the transfer matrix model for the FPMR system that is valid even in the limit of strong coupling when the coupled mode equations break down. We start with Fig. 6(a), which shows the microring resonator coupled to two waveguides of length \(L\) with partially reflective ends. The reflectivities at the ends of the top waveguide are \(r_1\) and \(r_2\), while for the bottom waveguide the reflectivities are \(r_1\) and \(r_1\). \(k_1\) and \(k_2\) are the fraction of the optical power coupled between the bottom and top waveguides, respectively, and the microring.

The evolution of the field amplitudes in the center microresonator is given by

\[
s' = e^{i\phi/2}u, \quad w' = e^{i\phi/2}v, \quad s = e^{i\phi/2}u', \quad w = e^{i\phi/2}v'
\]

where \(\phi = (\omega n/c + i\alpha/2)2\pi R\) is the propagation phase including losses for a wave of angular frequency \(\omega\) with RI \(n\).

The scattering matrix equations for the microring relate the fields both in the ring and in the waveguides that propagate toward each of the couplings \((a_1, a_2, w, s)\) at the bottom waveguide; \((c_1, c_2, w', s')\) at the top waveguide) to the fields

\[
\begin{align*}
\begin{bmatrix}
    a_1 \\
a_2 \\
w \\
s
\end{bmatrix} &=
\begin{bmatrix}
    k_1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & k_2 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    c_1 \\
c_2 \\
w' \\
s'
\end{bmatrix}, \\
\begin{bmatrix}
    a_1 \\
a_2 \\
w \\
s
\end{bmatrix} &=
\begin{bmatrix}
    1 & 0 & 0 & 0 \\
    k_1 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & k_2 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    c_1 \\
c_2 \\
w' \\
s'
\end{bmatrix}.
\]

Fig. 6. (a) Microring resonator evanescently coupled to two waveguides with partially reflective ends used in the derivation of transfer matrices. (b) The limit of the FPMR system obtained by setting the reflectivities of the lower waveguide to 0 and those of the upper waveguide to 1. The relevant traveling wave mode amplitudes used in the text are labeled in the figure.
propagating away from the coupling ($b_1, b_2, u, v$ at the bottom waveguide; $d_1, d_2, u', v'$ at the top waveguide). The coupling of the fields is expressible in terms of the scattering coefficients $t_1 = \sqrt{1 - k_i}$ and $k_i = i \sqrt{K_i}$, which are, respectively, the through coupling and coupling between the ring and the waveguide. For illustration, when $a_1 \neq 0$ and the other inputs are 0, the scattering equations are

$$v = t_1 w + k_1 a_1 e^{i \phi_b / 2}, \quad b_2 = t_2 a_1 e^{i \phi_b} + k_1 w e^{i \phi_b / 2}$$

$$v' = t_2 w', \quad d_1 = k_2 w' e^{i \phi_b / 2}$$

where $\phi = (\alpha n + \pi i/2) L$. These equations can be solved for $b_2$ and $d_1$ in terms of $a_1$, as

$$\frac{b_2}{a_1} = t_1 e^{i \phi_b} + \frac{t_2 k_2 e^{i \phi_b}}{e^{-i \phi_b} - t_2}, \quad \frac{d_1}{a_1} = k_1 k_2 e^{i \phi_b} e^{-i \phi_b / 2}$$

After proceeding in a similar manner for the other three input fields ($a_2, c_1$, and $c_2$), one obtains the transfer matrix describing the coupling between the waveguide fields and the microring as

$$\begin{bmatrix} b_2 \\ a_2 \\ c_2 \\ d_1 \end{bmatrix} = U_{MR} \begin{bmatrix} a_1 \\ c_1 \end{bmatrix},$$

where

$$U_{MR} = \begin{bmatrix} e^{i \phi_b (t_2 - t_1)} & 0 & 0 & e^{i \phi_b / 2} k_2 \\ 0 & 0 & 0 & 0 \\ -e^{i \phi_b / 2} k_2 & 0 & 0 & e^{i \phi_b (t_2 - t_1)} \\ -e^{i \phi_b (t_2 - t_1)} & 0 & 0 & -e^{i \phi_b / 2} k_2 \end{bmatrix}.$$

(A1)

The fields inside of the waveguides can now be related to the external fields at the reflective ends using the lossless transmission coefficient $T_2 = i \sqrt{1 - |r_2|^2}$. For example, at the left end of the lower waveguide, the field amplitudes are related by

$$a_1 = T_3 a_{in} - r_3 b_1, \quad b_{out} = T_3 b_1 - r_3 a_{in},$$

which after some rearrangement can be rewritten as the transfer matrix

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = U_3 \begin{bmatrix} a_{in} \\ b_{out} \end{bmatrix},$$

with

$$U_3 = \frac{1}{i \sqrt{1 - |r_3|^2}} \begin{bmatrix} -1 & -r_3 \\ r_3 & 1 \end{bmatrix},$$

with similar transfer matrices $U_i$ for the other reflecting ends ($i = 1, 2, 4$). The transfer matrix

$$U = \begin{bmatrix} U_4 & 0 & 0 & U_2 \\ 0 & U_4 & 0 & 0 \\ U_3 & 0 & U_3 & 0 \\ 0 & U_1 & 0 & U_1 \end{bmatrix}$$

relates the left fields $A_k^T = [a_{in}, b_{out}, c_{in}, d_{out}]^T$ to the right fields $A_k^T = [a_{add}, b_{through}, c_{add}, d_{through}]$ according to $A_k = U A_L$.

Finally, the transfer matrix for the FPMR sensor is obtained from $U$ by taking the limit $r_3, t_4 \to 0$ and $r_1, r_2 \to 1$ and equating $k_1 = K_{MW}$ and $k_2 = K_{MF}$.

REFERENCES

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