# The Role of Waveguide Coupling on the Sensitivity of a Coupled Resonator Optical Waveguide Gyroscope

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**Abstract:** We analyze the affect that the waveguide-resonator coupling, $\kappa_e$, has on the transmission and sensitivity of an optical gyroscope consisting of N evanescently coupled mircroring resonators. Rotation sensitivities are maximized as both $\kappa_e \longrightarrow 0$ and $\kappa_e \longrightarrow 1$.
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Coupled resonator optical waveguides (CROWs) are a linear array of identical optical micro-resonators coupled by nearest-neighbor evanescent fields [1]. In an idealized infinite CROW, the periodic structure leads to the creation of energy bands with group velocities proportional to the coupling between resonators. CROWs show significant promise for applications in integrated photonic circuits such as optical delay lines, buffers, and filters [1]. More recently, CROWs have been investigated for their potential as micro-optical gyroscopes with sensitivities exceeding MEMS gyros [2-4].

Finite length CROWs are coupled by evanescent coupling to input and output waveguides as illustrated in Fig. 1. The resonator-waveguide coupling, $0 \leq \kappa_e \leq 1$, represents the fraction of power coupled out of the resonator and is related to the decay rate of the intraresonator power $\kappa_e v_g/2\pi R$ where $v_g$ is the group velocity in the resonator of radius R. The CROW-waveguide coupling creates an effective Fabry-Perot (FP) resonator with finesse $F \approx \pi/\kappa_e$, which results in edge reflections that form ripples in the passband of the transmission spectrum referred to FP oscillations. Usually, the couplings are chosen simply to minimize these reflections [5]. This paper will show the importance of $\kappa_e$ and the FP oscillations for the sensitivity of a CROW gyroscope. We also show that the sensitivity of a CROW gyro increases as $(N+1)^2/\kappa_e$. Our calculations using transfer matrices [3] are based on linear propagation losses and resonator bending radii characteristic of silicon on insulator (SOI) and silicon oxynitride (SiON) CROWs [1].

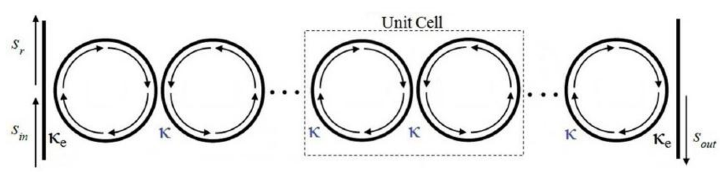


Fig. 1. Microring CROW gyroscope of N resonators coupled to waveguides. The transmitted field is $T(\phi_s) = |s_{out}/s_{in}|^2$ while the reflected field is $R(\phi_s) = |s_r/s_{in}|^2$. The inter-resonator coupling, $\kappa$, is defined similarly to $\kappa_e$ in the text.

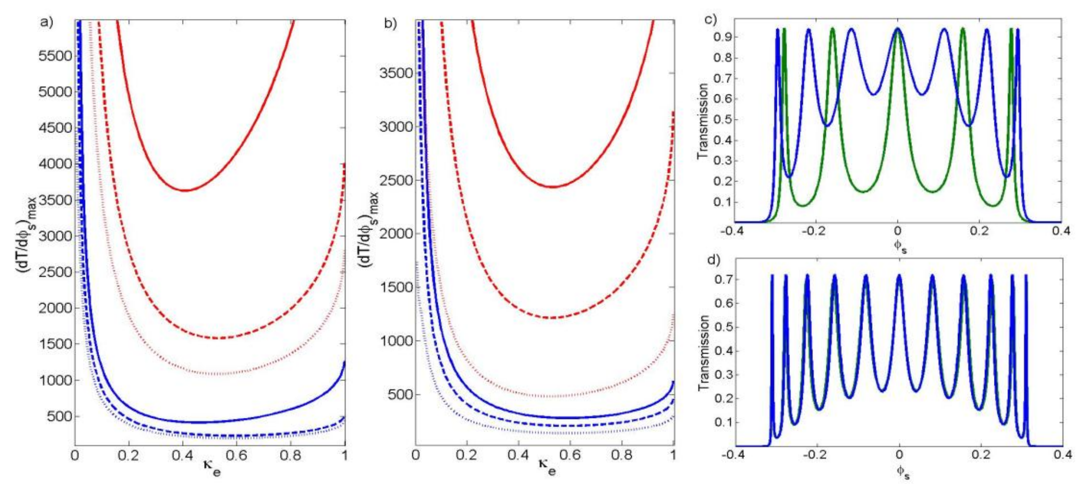


Fig. 2. Maximum transmission slope $(dT/d\phi_s)_{max}$ for N = 11(blue) and 23 (red) vs. $\kappa_e$ for losses of (a) $\alpha$ = 0 dB/cm (solid lines), 0.8 dB/cm (dashed lines), 1.5 dB/cm (dotted lines), and R = 10 $\mu$m. (b) $\alpha$ = 0 dB/cm (solid lines), 0.15 dB/cm (dashed lines), 0.35 dB/cm (dotted lines), and R=300 $\mu$m. Transmission with identical maximum slopes: (c) $(dT/d\phi_s)_{max}$ = 115.59 for N = 7 and $\kappa_e$ = 0.29 (blue) and $\kappa_e$ = 0.981 (green), where $\alpha$ = 1.5 dB/cm and R = 10$\mu$m. (d) $(dT/d\phi_s)_{max}$ = 439.05 for N = 11 with $\kappa_e$ = 0.1837 (blue) and N = 13 with $\kappa_e$ = 0.89 (green) for $\alpha$ = 0.15 dB/cm and R=300$\mu$m. In all cases $\kappa$ = 0.1. In (a) and (b) the CROWs are based on SOI while (b) and (d) are based on SiON CROWs.

When a CROW gyro, as depicted in Fig. 1, is rotated about an axis perpendicular to the plane of the device at a rate $\Omega$, the Sagnac effect leads to a nonreciprocal phase shift for light of frequency $\omega$ propagating in the clockwise and counterclockwise directions around each resonator. The roundtrip phase difference between the two directions is $2\phi_s=4\pi\omega\Omega R^2/c^2$. The sensitivity to changes in $\Omega$ is $dT/d\Omega=(2\pi\omega R^2/c^2)(dT/d\phi_s)$. The maximum value of $dT/d\phi_s$ is depicted in Fig. 2 (a) and (b) and is seen to be maximized both in the limit of $\kappa_e \longrightarrow 0$ (weak coupling regime) and $\kappa_e \longrightarrow 1$ (strong coupling regime) with a minimum sensitivity at an intermediate coupling.

In Fig. 2 (c) and (d), the maximum slope of $T(\phi_s)$ always occurs at the edges of the transmission band. As $\kappa_e \longrightarrow 0$, the depth of the FP transmission band ripples, whose number is N, increases similar to a FP resonator with increasing finesse. Furthermore, there are always two values of $\kappa_e$, one in the weak and one in the strong coupling regime, with identical slopes at the band edges as shown in Fig. 2(c). More generally, for a particular $\kappa_e$ in the strong coupling regime of an N+2 system, there exists a weak coupling $\kappa_e$ of an N resonator CROW gyro with an identical transmission and sensitivity as shown in Fig. 2(d).

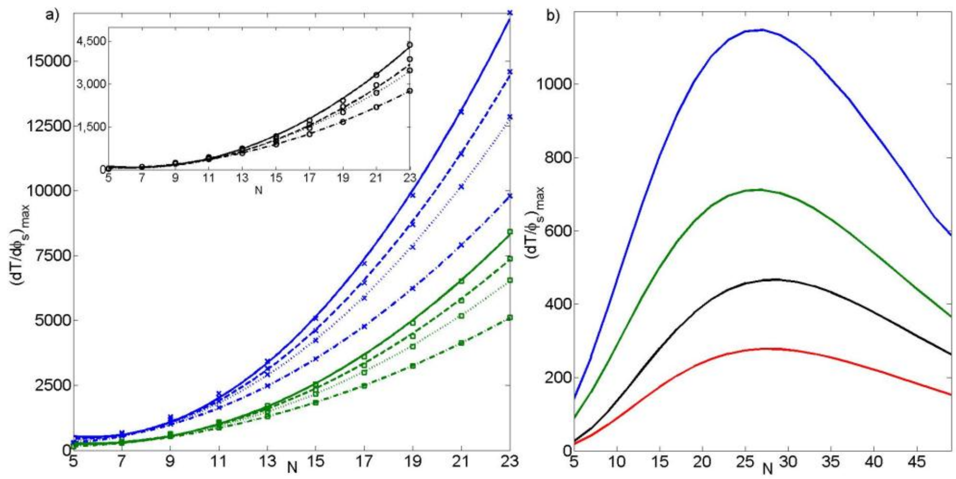


Fig. 3. $(dT/d\phi_s)_{max}$ vs. N in the weak coupling and strong coupling (inset) regimes. Data markers are numerically calculated slopes while lines are quadratic fits, $(dT/d\phi_s)_{max} = \alpha + \beta(N+1)^2$. Blue lines with crosses are $\kappa_e = 0.05$ while green lines with squares are $\kappa_e = 0.1$. Inset shows $\kappa_e = 0.95$. In all cases $\kappa = 0.1$ and R = 10$\mu$m resonators for $\alpha$=0 dB/cm (solid lines), 0.8 dB/cm (dashed lines) ,1.5 dB/cm (dotted lines) , and 3 dB/cm (dot dashed lines). (b) $(dT/d\phi_s)_{max}$ vs. N for $\alpha$=3 dB/cm showing a decrease in sensitivity for large N for $\kappa_e = 0.05$ (blue line), $\kappa_e = 0.1$ (green line), $\kappa_e = 0.1$ (red line), and $\kappa_e = 0.95$ (black line) all for $\kappa = 0.1$ and SOI CROWs.

In the weak coupling regime the maximum slope increases as $(dT/d\phi_s)_{max} \propto (N+1)^2/\kappa_e$, as shown in Fig. 3 when losses are small. However, in the presence of resonator losses, where $\alpha=(n\omega/c)Q_{int}^{-1}$ is the power attenuation per unit length in terms of the resonator's intrinsic Q-factor $Q_{int}$, signal attenuation leads to a lowering of the transmission slopes due to an exponential reduction in the transmission. In terms of the resonator mode number, $m=\omega nR/c$,

$$\left(\frac{dT}{d\phi_s}\right)_{\max} \propto \frac{(N+1)^2}{\kappa_e} e^{-\omega n L_{eff}/cQ_{\rm int}} = \frac{(N+1)^2}{\kappa_e} e^{-\left(\frac{N-1}{\kappa}+\frac{2}{\kappa_e}\right)\frac{m\pi}{Q_{\rm int}}}. \quad (1)$$

In conclusion, we have examined the effect of waveguide-resonator coupling, $\kappa_e$, on the sensitivity of a CROW gyroscope. By decreasing the coupling at the waveguide-CROW interface, the rotation sensitivity can be increased by a factor of $(N+1)^2/\kappa_e$ due to the narrowing of the FP oscillations. By contrast for strong coupling to the waveguides, the sensitivity is reduced because of not only broader FP oscillations but also because of a reduction in the effective enclosed geometric area of the gyroscope from $N(\pi R^2)$ to $(N-2)(\pi R^2)$.

## References


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