Effect of Static Disorder on Sensitivity of Coupled Resonator Optical Waveguide Gyroscopes

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Abstract—Coupled resonator optical waveguides (CROWS) have shown theoretical promise as microoptical integrated gyroscopes with sensitivities exceeding comparably sized MEMS gyroscopes. Here, we analyze the effect of static disorder on the rotational sensitivity of a CROW gyro. Since CROWS consist of a one-dimensional array of evanescently coupled ring-shaped microresonators, disorder includes random variations of the resonator optical path length and couplings between them resulting from manufacturing errors. We show that the rotational sensitivity of the CROW gyro is relatively stable with respect to variations in the interresonator coupling. By contrast, even modest fluctuations in the resonators’ roundtrip optical path length result in substantially lower rotational sensitivities. Finally, it is shown that while most random configurations of defects reduce the maximum sensitivity, there are particular configurations for which the rotation sensitivity is significantly higher than that of a CROW without disorder.

Index Terms—Gyroscopes, integrated optics devices, Sagnac effect.

I. INTRODUCTION

COUPLED resonator optical waveguides (CROWS) formed by an array of optical micro-resonators sequentially coupled by nearest-neighbor evanescent fields [1], have shown promise in a wide variety of applications such as optical delay lines, buffers, optical filters, and nonlinear optics [2]. While there are many forms of CROWS including photonic crystal defect cavities and coupled Fabry-Perot (FP) resonators, the most common type consists of micro-resonators formed by ring shaped integrated waveguides [3]. Recent theoretical work has shown that ring shaped micro-resonator CROWS hold great promise as micro-optical gyroscopes [4]–[10], with dimensions comparable to microelectromechanical systems (MEMS) gyroscopes, while having rotational sensitivities as much as several orders of magnitude better than MEMS and approximating those of much larger fiber optic gyroscopes (FOGs) [5], [7].

CROW gyroscopes are one of a number of proposed designs in recent years for chip based integrated micro-optical gyroscopes [11]–[14]. All of these diverse avenues of research in micro-optical gyroscopes are aimed at satisfying the growing need for highly sensitive portable gyro, which can not be satisfied by current commercial gyro scope technologies: Although FOGs [15] and ring laser gyroscopes (RLGs) [16] are able to achieve the necessary sensitivities for both inertial navigation and tactical applications they can not be integrated into small portable devices such as smartphones or drones because of their relatively large size and weight. By contrast, MEMS gyro have been able to reduce the overall footprint for integration onto standard semiconductor microchips but at the expense of sensitivity, usually two to four orders of magnitude worse than that of RLGs and FOGs [11].

To the best of our knowledge a CROW gyroscope has not yet been experimentally tested. While recent publications have shown CROW gyros to have sensitivities greatly exceeding commercial MEMS gyro [5], [7], these results are an idealization devoid of any realistic fabrication limitations. Although there has been substantial progress in understanding how the transmission and rotation sensitivity will be affected by resonator losses [9], [10], there has been no study of the effect that random disorder in the resonators, resulting from manufacturing errors, has on the transmission spectrum and ultimate rotation sensitivity of a CROW gyro. Disorder including both static [17], [18] and dynamic [19] fluctuations have been studied but only for non-gyroscopic CROWS. Understanding the effect of disorder, which can not be avoided in a multi-resonator structure, is important for comparing the practical potential of a CROW gyroscopes to other passive devices including a single integrated resonator of equal geometric area, which has already been shown to have superior sensitivity to the CROW when material losses are large such as in silicon on insulator (SOI) [9].

This manuscript explores the effect that static disorder will have on the achievable sensitivity, defined as the minimum detectable angular rotation rate, \( \Omega_{\text{min}} \). For circular micro-resonators, which we consider here, the disorder imposed by fabrication constraints are represented by variations in the resonator losses due to variations in the amount surface roughness from one resonator to another, and fluctuations in the evanescent coupling coefficient between the nearest neighbor resonator fields resulting from differences in the spacing between resonators. We show that the rotational sensitivity of the CROW gyro is much less affected by fluctuations of the resonator couplings than by fluctuations in the resonator path length. For example, fluctuations in the resonators’ radii of only several nm results in substantially lower rotational sensitivities on average by a factor of two or more with the reduction in average sensitivity being greater for smaller resonators. This is attributable to the change in the resonator mode frequencies, which depend on the roundtrip path length. Despite this...
reduced average sensitivity, there exist, however, particular configurations of the radius and coupling fluctuations for which the sensitivity is significantly higher than that of a CROW without disorder. This implies that while in the majority of instances a fabricated CROW gyro will have a sensitivity less than that of a perfect uniform CROW, in certain cases the fabricated CROW will have a sensitivity exceeding that of the uniform CROW.

II. MODEL

Fig. 1 provides a schematic of the CROW gyroscope indicating the relevant wave functions and couplings between resonators and waveguides. The CROW gyroscope consisting of a linear array of $N$ evanescently coupled circular microresonators of radius $R_j$ is coupled to input and output waveguides in an add-drop configuration. Rotations of this device about an axis perpendicular the plane of the device leads to a path length difference inside the individual rings proportional to the rotation rate, $\Omega$. For example a clockwise (CW) inertial rotation about an axis perpendicular to the plane of the resonators, results in an increased path length for the CW propagating wave, while simultaneously decreasing it for the counterclockwise (CCW) moving wave. Due to the change in the effective round-trip path length the CW wave acquires Sagnac phase $\phi_{S,j}$ while the CCW acquires the phase $-\phi_{S,j}$ upon one full revolution. The round-trip path difference between the two directions in the $j$th resonator is then $2\phi_{S,j} = 4\pi \omega \Omega R_j^2/c^2[15],[20]$.

To analyze the transmission through the rotating device we utilize the transfer matrix [21],[22] approach developed in Refs. [5],[7],[9] for rotating CROWs. In the CROW of Fig. 1, as a result of phase matching, light propagating in the CW direction of one resonator couples to the CCW mode of the adjacent resonator. The propagation of light in the CW and CCW directions and coupling to the nearest neighbor resonator are expressible in terms of transfer matrices $U_{CCW}^{(j)}$ and $U_{CCW}^{(j)}$, for the $j$th resonator, respectively

$$U_{CCW}^{(j)} = \frac{i}{\sqrt{\kappa_j}} \begin{bmatrix} \sqrt{1-\kappa_j}e^{i(\phi_j-\phi_{S,j})} & -e^{i(\phi_j+\phi_{S,j})} \\ e^{-i(\phi_j+\phi_{S,j})} & -\sqrt{1-\kappa_j}e^{-i(\phi_j-\phi_{S,j})} \end{bmatrix}.$$  \hspace{1cm} (2)

In addition, the coupling between the input waveguides and first resonator as well as the between the last resonator and output waveguide can be expressed in terms of the transfer matrices, $U_{in}$ and $U_{out}$, respectively [9]

$$U_{in} = \frac{-i}{\sqrt{\kappa_0}} \begin{bmatrix} \sqrt{1-\kappa_0}e^{i(\phi_1+\phi_{S,1})} & -e^{i(\phi_1+\phi_{S,1})} \\ e^{-i(\phi_1+\phi_{S,1})} & -\sqrt{1-\kappa_0}e^{-i(\phi_1+\phi_{S,1})} \end{bmatrix}.$$  \hspace{1cm} (3)

$$U_{out} = \frac{1}{\sqrt{\kappa_{N-1}\kappa_N}}$$

$$\times \begin{bmatrix} \sqrt{1-\kappa_{N-1}}e^{i(\phi_N+\phi_{S,N})} & -e^{i(\phi_N+\phi_{S,N})} \\ e^{-i(\phi_N+\phi_{S,N})} & -\sqrt{1-\kappa_{N-1}}e^{-i(\phi_N+\phi_{S,N})} \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 1 & -\sqrt{1-\kappa_{N-1}} \end{bmatrix}.$$  \hspace{1cm} (4)

The transfer matrix for a signal propagating from the in port to drop port of the output waveguide is expressible as

$$U = U_{out} \prod_{j=2}^{(N-1)/2} U_{CCW}^{(j)}(2) \prod_{j=2}^{(N-1)/2} U_{CCW}^{(j-1)}(2) \prod_{j=2}^{(N-1)/2} U_{CCW}^{(2)} = \left( U_{11} U_{12} \right) U_{CCW} U_{in} = \left( \begin{array}{cc} U_{11} & U_{12} \\ U_{21} & U_{22} \end{array} \right).$$

The transmission function

$$T = \frac{\left| U_{11} U_{22} - U_{21} U_{12} \right|^2}{\left| U_{22} \right|^2}$$

relates the output power in the drop port to the input power at the in port, $P_{out} = TP_{in}$. In the transfer matrices, $\phi_j = \beta_j \pi R_j$ is the propagation phase for the $j$th resonator with $\beta_j = \omega n_{eff}/f - i \alpha_j/2$ for light of angular frequency $\omega$ along with effective index of refraction $n_{eff}$ and linear power attenuation per unit length $\alpha_j$, [23]. $\kappa_j$ is the dimensionless coupling between the $(j-1)^{st}$ and $j$th resonators, which represents the fraction of the power coupled out of the $(j-1)^{st}$ resonator and into the $j$th resonator in each roundtrip such that $0 \leq \kappa_j \leq 1$ for $j = 1, \ldots, N - 1$. It is expressible in terms of an overlap integral between the mode functions in adjacent resonators and consequently depends on the cross-sectional dimensions of the resonators’ waveguides and the spacing between resonators [24]. Similarly, $\kappa_0$ and $\kappa_N$ are the couplings between the resonators and input and output waveguides, respectively.

We introduce disorder into our model by randomly varying the resonator radii, coupling coefficients, and losses:

$$R_j = R + \delta R_j,$$

$$\kappa_j = \kappa + \delta \kappa_j,$$

$$\alpha_j = \alpha + \delta \alpha_j.$$  \hspace{1cm} (5)

$R$, $\kappa$, $\alpha$ are the average values, which are the same for all resonators, while $\delta R_j$, $\delta \kappa_j$, $\delta \alpha_j$ are the random fluctuations, which have zero mean and are modeled using Gaussian probability distributions

$$P(\delta R_j) = \frac{1}{\sigma_R \sqrt{2\pi}} e^{-\delta R_j^2/2\sigma_R^2}.$$
where \(\sigma_R, \sigma_\kappa, \sigma_\alpha\) are the standard deviations of the fluctuations in radius, coupling coefficient, and losses, respectively. The resonance frequency of the \(m\)th axial resonator mode of the \(j\)th resonator is given by \(2\pi\omega_j n_{eff} R_j / c = 2\pi m\). It is assumed here that the optical source wavelength being used \(\lambda = 2\pi c / \omega = 1.55 \mu m\) is resonant with the average circumference of the resonators, \(2\pi R_{n_{eff}} = mL\). One must keep in mind that it is the optical path lengths of the resonators \(2\pi R_j n_{eff}\) that primarily controls the transmission of light through the CROW, which can fluctuate from one resonator to another due to both changes in \(R_j\) as well as the effective index, which arise from variations in the resonator waveguide shape or dimensions. However, since both effect the path length in the same manner, we have for the sake of simplicity in our model chosen only to model radius fluctuations while keeping the effective index constant. Moreover, the Sagnac phase \(\phi_{S,j}\) depends on \(R_j\) but not on the index of refraction and consequently, radius fluctuations are expected to have a larger impact on the gyroscope performance than the effective index.

In order to calculate the average transmission, we numerically perform an ensemble average by calculating the transmission for \(\ell = 1000\) trials. Each trial \(l\) consists of random values

\[
\begin{align*}
\{\delta R_1^{(l)}, \delta R_2^{(l)}, \delta R_3^{(l)}, \ldots, \delta R_N^{(l)}\}, \\
\{\delta \kappa_0^{(l)}, \delta \kappa_1^{(l)}, \delta \kappa_2^{(l)}, \delta \kappa_3^{(l)}, \ldots, \delta \kappa_N^{(l)}\}, \\
\end{align*}
\]

and \(\{\delta \alpha_0^{(l)}, \delta \alpha_1^{(l)}, \delta \alpha_2^{(l)}, \delta \alpha_3^{(l)}, \ldots, \delta \alpha_N^{(l)}\}\) for each of the \(N\) resonators chosen from the above Gaussian distributions from which the transmission spectrum of each trial, \(T^{(l)}\), is then calculated. The transmission for all \(\ell\) trials are then used to calculate the mean transmission according to

\[
\bar{T}(\phi_S) = \frac{1}{\ell} \sum_{l=1}^{\ell} T^{(l)}
\]

where \(\phi_S = 2\pi\omega\Omega R^2 / c^2\) is the average Sagnac phase shift in each resonator. Since the disorder is static and introduced during the fabrication, each trial may be considered to be a distinct fabricated device so that \(T^{(l)}\) would be the transmission from different devices while \(\bar{T}(\phi_S)\) would physically be the result of aggregating the transmission spectra from multiple devices.

The gyroscopic scale factor of each device, \(S^{(l)}\), gives the relative change in the output power due to a small change in the inertial rotation rate, \(\delta P_{out} / P_{in} = S^{(l)} \delta \Omega\), and is expressed as \([4, 5, 7, 9, 12]\)

\[
S^{(l)} = \frac{1}{P_{in}} \frac{dP_{out}}{d\Omega} = \frac{dT^{(l)}}{d\Omega} = \frac{1}{P_{in}} \frac{dP_{out}}{d\Omega} = -\frac{2\pi\omega R^2}{c^2} = \frac{dT^{(l)}}{d\phi}.
\] (6)

Here we define the sensitivity of the gyroscope to be the minimum detectable rotation rate \(\Omega_{\text{min}}\) such that greater sensitivity corresponds to smaller values of \(\Omega_{\text{min}}\) and vice versa. \(\Omega_{\text{min}}\) is found when the scale factor is a maximum such that \(S^{(l)}_{\text{max}}\) equals the relative noise fluctuations in the photodetector\([9, 11]\). In this study, three types of amplitude noise: quantum shot noise, thermal noise of the photodetector, and laser intensity noise are taken into account in order to calculate the minimum resolvable angular rate. We ignore phase noise due to the laser source linewidth under the assumption that the effective length of the CROW, \(L_{eff} = N\pi R / \kappa\) \([23]\) is less than the laser coherence length. The minimum detectable rotation rate of each device is then

\[
\Omega_{\text{min}}^{(l)} = \frac{1}{\Omega_{\text{max}}} \sqrt{\frac{2e}{\eta D} + \frac{4k_B T'}{R_L i_D} + RIN} \Delta f
\] (7)

where \(\eta = (e\bar{n} / \hbar)c P_{out}\) is the photodetector current, \(e\) is the fundamental electric charge, and \(\eta = 1\) the quantum efficiency of the photodetector. \(\Delta f = 1\) Hz is the measurement bandwidth, \(R_L = 50\) \(\Omega\) denotes the photodetector load resistance, \(T'\) is the temperature chosen to be 298 K, \(RIN\) is the relative intensity noise of the laser and was chosen to be \(-160\) dB/Hz \([11]\). All results here use \(P_{in} = 1\) mW, where shot noise is the dominant source of noise in the photodetector. By comparison, at input powers \(< 10\) \(\mu W\) thermal noise is the dominant noise while at input powers \(> 0.1\) W the system is dominated by laser noise. Lastly, the ensemble averaged sensitivity of all of the devices is given by

\[
\bar{\Omega}_{\text{min}} = \frac{1}{\ell} \sum_{l=1}^{\ell} \Omega_{\text{min}}^{(l)}.
\] (8)

CROWs based on various materials such as silicon oxyxynitride (SiON), Hydex glass, and SOI have been routinely fabricated and demonstrated for applications in integrated photonic circuits \([2]\). Despite the popularity of SOI for integrated optical micro-resonators, the large losses, \(\alpha = 1 - 3\) dB/cm \([25]\), have been shown to severely limit the rotational sensitivity of an SOI based CROW gyro \([9]\) and for that reason we do not consider SOI in this work. For our analysis we assume that the CROWs are fabricated from a low index material such as SiON or Hydex. SiON has an index of refraction \(n = 1.513\) and in index contrast between resonator waveguide and cladding of \(\Delta n_{\text{SiON}} = 4.5\%\) corresponding to a minimum bending radius of 300 \(\mu m\) along with losses of about \(\alpha_{\text{SiON}} = 0.15 - 0.35\) dB/cm \([26]-[28]\). Hydex glass with refractive index \(n \approx 1.5 - 1.9\) has by far the smallest losses \(\alpha_{\text{Hydex}} = 0.06\) dB/cm with an index contrast between waveguide and cladding of \(\Delta n_{\text{Hydex}} = 17\%\) leading to a minimum bending radius of 40 \(\mu m\) \([29, 30]\).

### III. Results

In order to isolate the effects of both types of disorder, we separately consider variations in the radii and coupling coefficients in the following two subsections. Also, in these subsections we focus on idealized lossless resonators, \(\alpha_j = 0\), with an effective index of refraction \(n_{eff} = 1.5\). Later in Subsection III-C, we will include nonzero losses and specific effective indices for SiON and Hydex. Moreover, in all simulations in this paper we chose the average total geometric area of the \(N\) resonator
A. Effect of Fluctuations of Resonator Radii

To start, we consider fluctuations in the resonator radii for uniform inter-resonator couplings ($\delta R_j = 0$) and no losses ($\alpha_j = 0$). A typical tolerance of an electron beam lithography system such as the JEOL 6300FS [34] is several nanometers in terms of positional accuracy within the field $\leq \pm 9$ nm where it is safely assumed that the percentage outside the confidence interval is 0.001% or $4.4\sigma$ and as a result we use $\sigma_R \approx 2$ nm. Fig. 2 shows the transmission spectra for $N = 5$ and 15 each showing three different trials as well as the ensemble averaged mean transmission for $\sigma_R = 2$ nm. For reference, the transmission of a uniform CROW ($\sigma_R = 0$) is also shown.

One can see that the variations in transmission from one device trial to another are substantial in both cases as exhibited by the shifting locations of the $N$ FP resonances in the transmission band for each device. The effect of the variations is manifest in the averaged transmission, which is significantly reduced. These FP resonances are the result of the impedance mismatch between the waveguides and resonator array causing the entire CROW array to behave as an FP resonator [8]. Moreover, because the FP transmission resonances become narrower for increasing $N$, the reduction of the average transmission becomes greater as $N$ increases owing to the decreasing likelihood of FP resonance locations from different trials overlapping. The reason for the shifting FP resonance locations are the fluctuations in the resonance frequency of the resonators $\omega_j$, which creates a randomly varying detuning $\Delta_j = \omega - \omega_j$ throughout the array. These detunings change the roundtrip optical phase within the array, which determines the locations of the FP resonances.

Fig. 3 shows the average values and standard deviations of $\Omega_{\text{min}}$ for $\sigma_R = 1, 2, 3$ and 6 nm as a function of $N$ compared to $\Omega_{\text{min}}$ of the uniform device for which $\sigma_R = \sigma_R = 0$. The top of each graph shows on the horizontal axis the average resonator size $R$ as $N$ increases. For the uniform CROW gyro without losses it is known that $\Omega_{\text{min}}$ decreases monotonically with increasing $N$ [4], [8], [9]. However, the ensemble average $\Omega_{\text{min}}$ decreases much more slowly with $N$ while the standard deviation increases with $N$ both of which are indicative of the larger device to device variations in the scale factor. The relative fluctuations of each resonator, $\sigma_R/R$, become larger with $N$. As a result, the relative fluctuations in the sensitivity $\sigma(\Omega_{\text{min}})/\Omega_{\text{min}}$ also grow as $N$ increases. In fact, one can see that for $\sigma_R = 6$ nm and $N > 11$ there is an order of magnitude reduction in the average sensitivity compared to the uniform CROW while $\sigma(\Omega_{\text{min}})/\Omega_{\text{min}} \sim 1$. This can be considered a practical upper limit on the tolerable disorder.

At this point it is worth emphasizing that the ensemble averaged transmission and sensitivity, which represents the aggregate behavior of many slightly different gyros is in reality a poor indicator of the behavior of individual gyros all of which have

CROW, $N(\pi R^2)$, to be 4 mm$^2$, roughly the size of a typical MEMS gyro. This implies that that as the number of resonators increases, their radii decreases. We use an average coupling of $\kappa = 0.1$ in all simulations, which is consistent with the range of values used in experiments with SiON and Hydex CROWs having fabricated couplings ranging from $\kappa = 0.05$ to 0.8 [27], [31]–[33].
Fig. 4. Histogram of occurrences of $\Omega_{\min}$ for an ensemble of 10,000 trials for a) $N=5$ ($R=447.7 \mu m$) and b) $N=15$ ($R=258.7 \mu m$) resonators with $\sigma_R = 2 \mu m$ and $\sigma_\alpha = 0$. The green highlighted bar shows $\Omega_{\min}$ for a uniform device while the red bar shows the ensemble averaged sensitivity $\bar{\Omega}_{\min}$. Each bar corresponds to a range of 4.5 deg/hr. Here $\sigma_\alpha = \alpha = 0$ and $n_{\text{eff}} = 1.5$.

B. Effect of Fluctuations of Inter-Resonator Coupling

Next we consider the effect of variations in the coupling for resonators with identical radii ($\delta R_j = 0$) and no losses ($\alpha_j = 0$). Fig. 5 again shows the transmission spectra for $N=5$ and 15 for a 5% variation in the couplings. One can see that the differences in the locations of the FP resonances for different devices is significantly smaller than was the case for variations in the radii. Consequently, the average transmission is much larger than in the case of radii fluctuations.
Fig. 6. Ensemble averaged sensitivity $\Omega_{\min}$ versus $N$ (black lines corresponding to left y-axis) and standard deviation of the sensitivity $\sigma(\Omega_{\min})$ versus $N$ (red lines with diamonds corresponding to right y-axis) for $\sigma_\kappa/\kappa = 0$ (blue dash dot line), $\sigma_\kappa/\kappa = 0.01$ (solid lines), $\sigma_\kappa/\kappa = 0.05$ (dashed lines), and $\sigma_\kappa/\kappa = 0.1$ (dotted lines). In all cases $\sigma_R = 0$, $\sigma_\alpha = \alpha = 0$, and $n_{eff} = 1.5$.

Fig. 7. Histogram of occurrences of $\Omega_{\min}$ for an ensemble of 10,000 trials for a) $N = 5$ ($R = 447.7 \mu m$) and b) $N = 15$ ($R = 258.7 \mu m$) resonators with $\sigma_\kappa/\kappa = 0.05$. The red highlighted bar shows $\Omega_{\min}$ for a uniform device and the green highlighted bar shows the ensemble averaged mean sensitivity $\Omega_{\min}$. Each bar corresponds to a range of $1$ deg/hr for $N = 5$ and $1.7$ deg/hr for $N = 15$. For $N = 5$, $\Omega_{\min} = 77$ deg/hr, which is equal to $\Omega_{\min}$ of the uniform device corresponding to the same interval of the histogram. Here $\sigma_\alpha = \alpha = 0$ and $n_{eff} = 1.5$.

Fig. 8. Ensemble averaged sensitivity $\Omega_{\min}$ (black lines- left y-axis) and standard deviation $\sigma(\Omega_{\min})$ (red lines with diamonds- right y-axis) versus number of resonators, $N$ for a) $\sigma_\kappa/\kappa = 0.05$ and b) $\sigma_\kappa/\kappa = 0.1$. Additionally $\sigma_R = 2 \mu m$ (solid lines), $\sigma_R = 4 \mu m$ (dashed lines), $\sigma_R = 6 \mu m$ (dotted lines). The blue dash dot line is for a disorderless uniform CROW. In all simulations $\sigma_\alpha = \alpha = 0$ and $n_{eff} = 1.5$.

Fig. 9. Transmission spectra versus average Sagnac phase $\phi_S$ of 3 disordered CROWs (solid lines) compared to the ensemble averaged transmission (dashed line) and that of a uniform device (dash dot line) for a) $N = 5$ ($R = 447.7 \mu m$) and b) $N = 11$ ($R = 302 \mu m$). The variations of the coupling and radii are zero, $\sigma_\kappa = \sigma_R = 0$ while the loss variations are non-zero, $\sigma_\alpha/\alpha = 1$. The simulation corresponds to SiON with $n_{eff} = 1.505$ and $\alpha = 0.15$ dB/cm.
fluctuations. This indicates that the variations in the optical path length are significantly more detrimental to the sensitivity than the coupling variations.

C. Effect of Losses on the Disordered Sensitivity

Lastly, we consider the effect of nonzero resonator losses, \( \alpha_j \neq 0 \). To do so, in Figs. 9–11 we specifically consider SiON with \( n_{\mathrm{eff}} = 1.505 \) and \( \alpha = 0.15 \) dB/cm [36] as well as Hydex glass with \( n_{\mathrm{eff}} = 1.7 \) and \( \alpha = 0.06 \) dB/cm [32]. To first examine the effect of fluctuations in the losses resulting from variations in the surface roughness of the resonators, we plot in Fig. 9 the transmission through \( N = 5 \) and 11 SiON CROWs with \( \delta R_j = \delta \kappa_j = 0 \) and \( \sigma_\alpha / \alpha = 1 \). This level of fluctuations in the losses is much larger than what is in practice observed. However, as one can see in Fig. 9, the transmission is barely affected by the loss fluctuations except for a small diminishment in the peak transmission. If we assume that the light circulates around each resonator 1/2\( \pi \) times, then the net signal attenuation in each resonator is \( \exp(-\alpha_j \pi R / \kappa) \) [8], [23]. The total attenuation of the CROW is then \( \exp(-\pi R / \kappa \sum \alpha_j) = \exp(-\pi R / \kappa N \alpha) \) where we have used the fact that \( \delta \alpha_j \) has zero mean. Thus in the case of uniform radii and coupling, the net loss of the CROW only depends on the average resonator loss.

Next we examine the combined effect of radius and coupling disorder in the presence of constant propagation losses, \( \delta R_j, \delta \kappa_j \neq 0 \) but \( \delta \alpha_j = 0 \). In Fig. 10 we show again the transmission for SiON and Hydex and with fluctuations of \( \sigma_\alpha = 2 \) nm and \( \sigma_\kappa / \kappa = 0.1 \). The losses ameliorate the effect of the disorder on the transmission with the variations between devices being reduced compared to Fig. 2 although the ensemble averaged transmission is similar to Fig. 2 but now being due to the non-zero losses. In Fig. 11 one can see the average sensitivity \( \Omega_{\text{min}} \) for both types of disorder and non-zero propagation losses. In comparison to Figs. 3 and 6, one can conclude that propagation losses have a much larger effect on the average sensitivity than disorder with \( \Omega_{\text{min}} \) actually increasing for \( N \gtrsim 5 \). By contrast, the standard deviation of \( \Omega_{\text{min}} \) in Fig. 11 is only slightly larger than in Fig. 3 indicating that the losses do not contribute to the device to device fluctuations as one would expect. (The increase in \( \sigma(\Omega_{\text{min}}) \) compared to Fig. 3 is due to the inclusion of fluctuation in the couplings as well as radius in Fig. 11.)

IV. DISCUSSION AND CONCLUSION

It is clear from the above analysis that variations in the size of the resonators is the most pernicious form of disorder due to the dependence of the resonators’ mode frequencies on the circumference. In order to limit the effect of this form of disorder, the standard deviation of the fluctuations of the resonator frequencies would have to satisfy \( \sigma_\omega / \omega_0 < \Delta \omega / \omega_0 = Q^{-1} \) where \( \omega_0 = mc / Rn \) is the average mode frequency, \( \Delta \omega \) is the resonator bandwidth, and \( Q \) is the loaded Q-factor of the
resonator, which is typically $\sim 10^3 \sim 10^4$. For small fluctuations, $\sigma_R/\omega_0 \approx \sigma_R/R$ requiring the relative size fluctuations to be $\sigma_R/R \ll 10^{-3}$, which is consistent with the values of $\sigma_R$ used here. By contrast, fluctuations in the coupling merely modulate the resonator bandwidths resulting in a slight broadening of the FP transmission band resonances.

Overall though, the effect of disorder on the sensitivity is small compared to ordinary propagation losses in commonly used waveguide materials. Variations in the resonator waveguide width as well as the sidewall roughness could be reduced by using a high quality photosensitive like HSQ for the lithography and etching processes thereby reducing the overall variations in the optical path length as well as losses due to scattering. Also, as has been demonstrated experimentally, electrical heaters fabricated above the resonators can be used to control the index of refraction of the resonator by the thermo-optic effect, which can be used to compensate for any fabrication induced variations in the resonators’ optical path length [17], [25]–[27].

It is important to emphasize that the static disorder studied here would be introduced during the fabrication of individual devices such that each device would have a unique pattern of disorder with its own characteristic transmission spectrum and rotational sensitivity different from the average values obtained from an ensemble of separate devices. Although the locations of the transmission band resonances (and equivalently the location of maximum scale factor) shift from one device to another, these differences could be calibrated by using appropriate phase biasing. Even though each device would then have to be individually calibrated, once calibrated though a significant minority of the devices will have sensitivities better than that of an ideal uniform CROW.

Finally, it should be pointed out that for very large amounts of disorder, particularly with respect to the resonator path lengths $Q^{-1} \ll \sigma_R/R$, the transmission through the CROW would be heavily suppressed. In this case an integrated gyroscope consisting of a single resonator with an enclosed area equal to the CROW would have superior sensitivity. This is consistent with results in Ref. [9] that showed that the sensitivity of the single resonator begins to surpass the CROW’s when the losses start to exceed 0.1 dB/cm.

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Authors’ biographies not available at the time of publication.