Effect of resonator losses on the sensitivity of coupled resonator optical waveguide gyroscopes

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Recently there has been a growing interest in microphotonic integrated optical gyroscopes. Here, we analyze the effect of resonator losses on the rotational sensitivity of a coupled resonator optical waveguide (CROW) gyroscope in comparison to a single passive resonator gyroscope of the same size. We show that the CROW gyro offers a superior sensitivity only for very low propagation losses. Moreover, the single ring resonator gyro is found to have a sensitivity that is stable over wide range of resonator losses as well as boasting greater sensitivities than the CROW gyro for propagation losses in the resonators exceeding 10^{-1} dB/cm.

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In recent years, there has been a growing interest in micrometer-scale integrated optical gyroscopes. A variety of different designs for both active and passive integrated optical gyroscopes have been proposed [1,2]. Such microphotonic gyroscopes have dimensions comparable to MEMS gyroscopes with theoretical sensitivities significantly exceeding MEMS. Moreover, microphotonic gyroscopes, like MEMS, can be fabricated using standard semiconductor fabrication techniques but, unlike MEMS, they have no moving parts making them easier to fabricate and more robust. Such devices may potentially satisfy the need for low cost on-chip gyroscopes for high-sensitivity applications such as hand-held navigation in GPS denied regions [1].

Among the different types of integrated optical gyros that have been proposed, passive linear arrays of evanescently coupled optical microresonators known as coupled resonator optical waveguides (CROWs) [3] have shown theoretical promise as gyroscopes [4–11]. However, there has also been much debate as to the actual sensitivity of a CROW gyroscope with some works claiming that they are no more sensitive than a single passive resonator of equal geometric area as the CROW [5,6], while other work either explicitly or implicitly claims sensitivity exceeding that of a single resonator of equal area [4,7–9]. The issue is complicated by some work using modified forms of CROWs specifically to enhance the sensitivity [7,8] while other work provides few details of the mathematical model used to evaluate the sensitivity [5]. Also, previous works have focused largely on lossless resonators ignoring the effects of propagation losses in the resonator, which we will show here to be decisive in resolving the question of which device—CROW or individual resonator—has the greatest sensitivity.

The goal of this publication is to clarify the two seemingly contradictory stances regarding the CROW gyro's sensitivity compared to that of an equivalent individual resonator of the same footprint. This is accomplished by evaluating the maximum achievable sensitivity, defined as the minimum detectable angular rotation rate of a CROW gyro consisting of N coupled circular resonators and a single ring resonator gyro of equal size for various propagation losses, geometric constraints, and coupling coefficients. We show that CROW gyros offer a sensitivity enhancement over the equivalent single resonator gyro only when propagation losses are small. Moreover, the single resonator gyro has sensitivity that is stable over a wider range of losses while also achieving greater sensitivity at losses typical of many commonly used materials. Note that although [4] and [10] considered losses in a CROW gyroscope, their work did not consider the effect of losses on the achievable sensitivity.

The CROW gyroscope, illustrated in Fig. 1, consists of a linear array of N evanescently coupled circular ring shaped microresonators of radius R coupled to input and output waveguides. When the CROW is rotated about an axis perpendicular to the plane of the device, the Sagnac effect leads to a nonreciprocal phase shift for waves of frequency ω propagating in the clockwise (CW) and counterclockwise (CCW) direction around each resonator. Assuming a CW inertial rotation Ω, the round-trip path length would increase slightly for a CW wave, while simultaneously decreasing for a CCW wave. The CW wave acquires upon one full revolution around a resonator the Sagnac phase ϕ_S, while the CCW acquires the phase $-\phi_S$ due to the change in the effective round-trip path length where $\phi_S = 2\pi\omega\Omega R^2/c^2$ [1].

To analyze the transmission through the rotating CROW we utilize the transfer matrix [12] approach developed in [7,8] for rotating resonators. Transmission can be

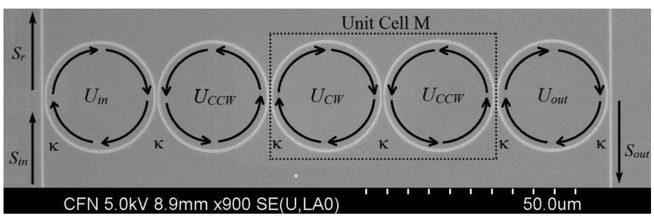

Fig. 1. SEM image of a CROW fabricated on SOI, showing the relationship between the resonators and transfer matrices defined in text. $s_{\rm in}$ and $s_{\rm out}$ are the input and transmitted fields such that the transmission is $T(\phi_S) = |s_{\rm out}/s_{\rm in}|^2$. Based on the propagation direction of $s_{\rm out}$, the number of resonators must be odd due to phase matching.

expressed in terms of input (U_{in}) and output (U_{out}) matrices for the first and last resonators coupled to the waveguides as well as U_{CCW} and U_{CW} for the repetitive unit cell of the array. Due to phase matching, light in resonators with U_{CCW} propagates in the CCW direction while in the adjacent resonators represented by U_{CW} the signal propagates in the CW direction:

$$U_{\text{in}} = \frac{-i}{\sqrt{\kappa}} \begin{pmatrix} \sqrt{1-\kappa_e} e^{i(\phi_p+\phi_S)} & -e^{i(\phi_p+\phi_S)} \\ e^{-i(\phi_p+\phi_S)} & -\sqrt{1-\kappa_e} e^{-i(\phi_p+\phi_S)} \end{pmatrix}, \quad (1)$$

$$U_{\text{out}} = \frac{1}{\kappa} \begin{pmatrix} \sqrt{1-\kappa_e} e^{i(\phi_p+\phi_S)} & -e^{-i(\phi_p+\phi_S)} \\ e^{i(\phi_p+\phi_S)} & -\sqrt{1-\kappa_e} e^{-i(\phi_p+\phi_S)} \end{pmatrix} \times \begin{pmatrix} \sqrt{1-\kappa} & -1 \\ 1 & -\sqrt{1-\kappa} \end{pmatrix}, \quad (2)$$

$$U_{\text{CW}} = \frac{-i}{\sqrt{\kappa}} \begin{pmatrix} \sqrt{1-\kappa} e^{i(\phi_p+\phi_S)} & -e^{i(\phi_p+\phi_S)} \\ e^{-i(\phi_p+\phi_S)} & -\sqrt{1-\kappa} e^{-i(\phi_p+\phi_S)} \end{pmatrix}, \quad (3)$$

$$U_{\text{CCW}} = \frac{i}{\sqrt{\kappa}} \begin{pmatrix} \sqrt{1-\kappa} e^{i(\phi_p-\phi_S)} & -e^{i(\phi_p-\phi_S)} \\ e^{-i(\phi_p-\phi_S)} & -\sqrt{1-\kappa} e^{-i(\phi_p-\phi_S)} \end{pmatrix}. \quad (4)$$

Here, $\phi_p = \beta\pi R$ and $\beta = \omega n/c - i\alpha/2$ with effective index of refraction n. α is the linear power attenuation coefficient per unit length in the resonators. κ is the dimensionless coupling between resonators, $0 \leq \kappa \leq 1$, which represents the fraction of the power coupled out of a resonator in each round trip, while κ_e is the analogous coupling to the waveguides for which we take $\kappa = \kappa_e$ here. The transmission matrix for a signal propagating from the left to right waveguide is given as

$$T_N = U_{\text{out}}(U_{\text{CCW}}U_{\text{CW}})^M U_{\text{CCW}}U_{\text{in}} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix},$$

where $M = (N-3)/2$. The transmission, which is proportional to the output power, is $T(\phi_S) = |T_{11}T_{22} - T_{12}T_{21}|^2/|T_{22}|^2$. The single resonator gyro is just $N = 1$.

The gyroscope scale factor, S, which measures the sensitivity to rotations, relates the change in the transmission to a small change in the inertial rotation rate, $\delta T = S\delta\Omega$, and is expressed as [1,5–8]

$$S = \frac{1}{P_{\text{in}}}\frac{dP_{\text{out}}}{d\Omega} = \frac{dT}{d\Omega} = \left(\frac{2\pi\omega R^2}{c^2}\right)\left(\frac{dT}{d\phi_S}\right), \quad (5)$$

where $P_{\text{out}} = TP_{\text{in}}$ is the optical power at the device output in terms of the input power. In this study, quantum shot noise, thermal noise of the photodetector, and laser intensity noise are taken into account in order to calculate the minimum resolvable rotation rate. We ignore phase noise under the assumption that the effective length of the CROW L_{eff} is less than the laser source coherence length. The effective length $L_{\text{eff}} = \pi RN/\kappa$ is equal to the total circumference of the resonators, $N(2\pi R)$, times the average number of times a photon circulates in each resonator, $1/2\kappa$ [13]. The minimum detectable rotation Ω_{min} is found when the scale factor is a maximum such that $S_{\text{max}}\Omega_{\text{min}}$ equals the relative noise fluctuations in the photodetector [14],

$$\Omega_{\text{min}} = \frac{1}{S_{\text{max}}}\sqrt{\left(\frac{2e}{i_D} + \frac{4k_B T'}{R_L i_D^2} + RIN\right)\Delta f}, \quad (6)$$

where $i_D = (e\eta\lambda/hc)P_{\text{out}}$ is the photodetector current, e is the fundamental electric charge, and $\eta = 1$ the quantum efficiency of the photodetector. $\Delta f = 1$ Hz is the measurement bandwidth, $R_L = 50\Omega$ denotes the photodetector load resistance, T' is the temperature chosen to be 298 K, RIN is the relative intensity noise of the laser and was chosen to be −160 dB/Hz [14]. All results here use a wavelength of $\lambda = 1.55$ μm and $P_{\text{in}} = 1$ mW, where shot noise is the dominant source of noise. In the discussion that follows, the gyroscope sensitivity is defined to be Ω_{min} such that smaller values of Ω_{min} correspond to better sensitivities and vice versa.

First, a comparison of the sensitivities of a CROW with N resonators each with the same fixed radius and a single resonator with the same circumference as the CROW, $N(2\pi R)$. Figure 2(a) shows the lossless case, $\alpha = 0$. One can see that the CROW has a greater sensitivity (i.e., smaller Ω_{min}) than the single resonator when $N \gg 1$ while for small N, the single resonator has better sensitivity. For example, an $N = 7$ CROW with $R = 500$ μm has a sensitivity of $\Omega_{\text{min}} = 36$ deg/hr, while a single resonator with a radius of 3500 μm will produce $\Omega_{\text{min}} = 21$ deg/hr, almost twice the sensitivity of the CROW. By contrast, for $N = 75$ each with $R = 100$ μm, the sensitivity is $\Omega_{\text{min}} = 1.2$ deg/hr, while a single resonator with a radius of 7500 μm will have $\Omega_{\text{min}} = 4.5$ deg/hr, roughly 4 times worse than the CROW.

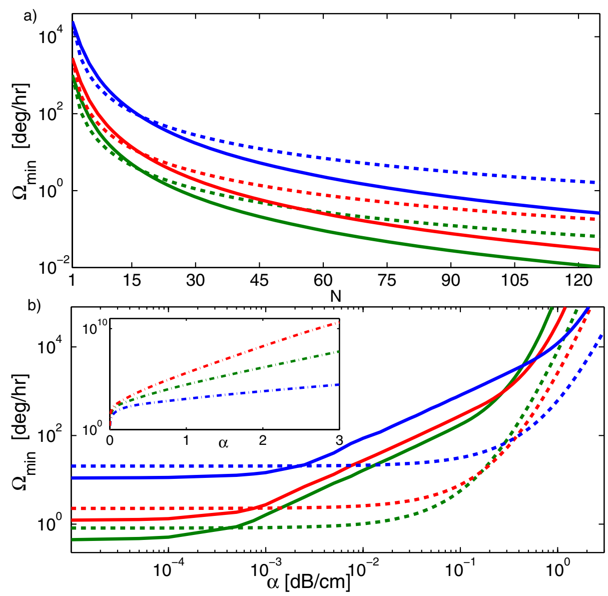

Fig. 2. (a) Ω_{min} versus N for lossless resonators and (b) Ω_{min} versus propagation loss, α, for $N = 35$. Inset shows the ratio of Ω_{min} with and without losses for the CROW gyro. Solid lines represent CROWs while dashed lines are Ω_{min} of a single ring resonator with circumference equal to that of the CROW, $N(2\pi R)$ both for $\kappa = 0.1$. The CROW radii are $R = 100$ μm (blue lines), 300 μm (red lines), 500 μm (green lines).

Figure 2(b) examines the effect of propagation loss on the sensitivity of the CROW gyro and single resonator of equal total circumference. A 35 resonator CROW was chosen since this value of N exhibited a significant improvement in sensitivity over the single resonator structure for $\alpha = 0$ for all R values tested. For low losses, $\alpha \lesssim 10^{-3}$ dB/cm, the CROW yields greater sensitivity, approximately twice as good as the equivalent single resonator. However, the single resonator sensitivity is much more stable with respect to increases in α up to nearly $\alpha = 0.1$ dB/cm after which the sensitivity rapidly degrades. By contrast the sensitivity of the CROW starts to quickly deteriorate for losses above 10^{-3} dB/cm, becoming significantly worse than the single resonator sensitivity by as much as a factor of 10^2. For example, the sensitivities of CROW gyros with $R = 100$ μm and 500 μm decrease by a factor of 10 compared to the lossless case when $\alpha = 0.0125$ and $\alpha = 0.026$ dB/cm, respectively. By contrast the single resonator gyros with equal circumferences to the CROWs exhibit the same factor of 10 decrease in sensitivity compared to the lossless case when $\alpha = 0.61$ and 0.125 dB/cm, respectively.

Next we consider a CROW confined to a fixed geometric area and compare it to a single resonator with the same area, $N(\pi R^2)$. Areas were chosen to be 4 mm^2, 20 mm^2, and 50 mm^2, which are comparable to commercially available MEMS gyros. Note that the total area of the CROW is fixed, implying that R decreases with increasing N. Figure 3(a) shows the sensitivity of the CROW and single resonator of equal area for $\alpha = 0$ where one sees that $\Omega_{\min}$ is inversely proportional to the geometric area of both devices as expressed in Eq. (5). Note the sensitivity of the single resonator is always worse than the CROW for all N. For the CROW, maximizing N by using the smallest possible resonators (determined by the minimum bending radius of the material) leads to the best sensitivity for the CROW. In fact, a numerical data fit for the values of $S_{\max}$ versus N shows that for $N \gg 1$, $S_{\max} \propto N^4$ when $\alpha = 0$. Physically this is a result of the sharpening of the slope of the CROW transmission band edges as N increases.

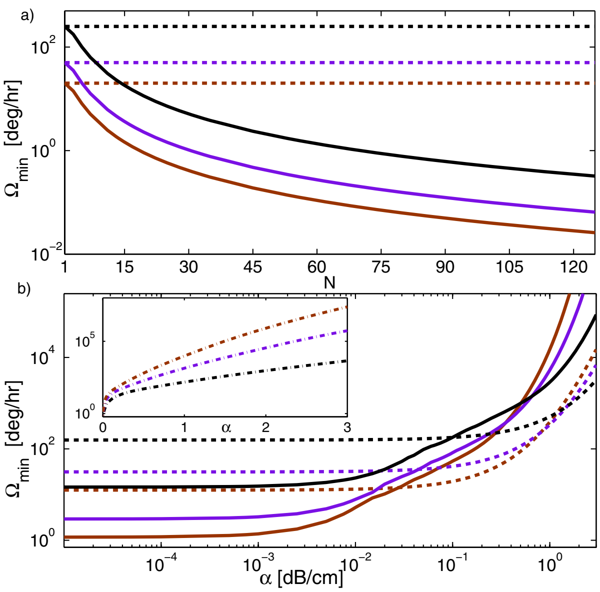

Fig. 3. (a) $\Omega_{\min}$ versus N for lossless resonators and (b) $\Omega_{\min}$ versus α for $N = 17$ constrained to an area of 4 mm^2 (black lines), 20 mm^2 (purple lines), and 50 mm^2 (brown lines). Inset shows the ratio of $\Omega_{\min}$ with and without losses for the CROW gyro. Solid lines are CROW gyros and dashed lines are single resonators of equal area both for $\kappa = 0.1$.

Figure 3(b) considers propagation losses for the two devices now with fixed areas. Again, for low losses, $\alpha < 10^{-2}$ dB/cm, the $N = 17$ CROW yields a nearly order of magnitude better sensitivity than the single resonator of equal area. However, the single resonator gyro sensitivity is once again more stable with respect to increasing α over a much wider range as well as boasting greater sensitivities for $\alpha \gtrsim 0.1$ dB/cm than the equivalent CROW device. For example, when confined to an area of 20 mm^2, the sensitivity of a 17 resonator CROW becomes an order of magnitude worse than for no losses when $\alpha = 0.038$ dB/cm, while the sensitivity of a single resonator of the same area becomes an order of magnitude worse than the lossless case when $\alpha = 0.98$ dB/cm. Similarly, an $N = 17$ CROW and a single resonator gyro of equal area of 50 mm^2 with $\alpha = 0.3$ dB/cm produces $\Omega_{\min} = 184$ and 58 deg/hr, respectively.

To understand the superior performance of the single resonator for large losses, first recall that the transmission of the CROW as a function of ϕ_S consists of a transmission band of N closely spaced transmission resonances, as shown in Fig. 4. For a lossless CROW, the maximum slope of the transmission always occurs at the edges of the transmission band and yields the maximum scale factor $S_{\max}$. However, for $\alpha \neq 0$, signal attenuation leads to a reduction in amplitude of each of the resonances. Moreover, the transmission resonances at the band edges are significantly more reduced as α increases than is the central resonance. Consequently, as α increases, the maximum transmission slope shifts from the outermost resonances toward the central resonance, as depicted in Fig. 4. The shift in the location of the maximum transmission slope from the edge to the central resonance corresponds to the abrupt reduction in sensitivity seen in Figs. 2(b) and 3(b) as α is increased. By contrast, the single resonator transmission exhibits only a single

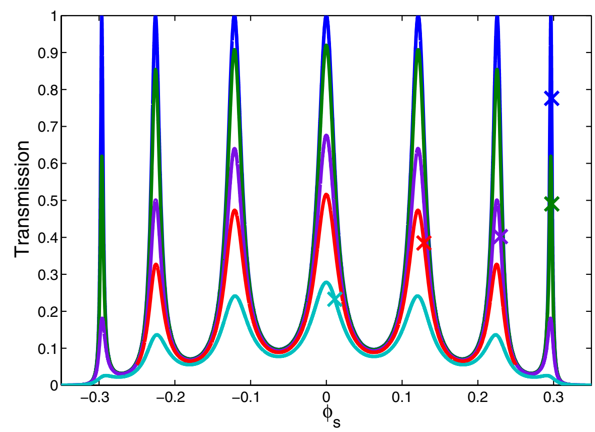

Fig. 4. Transmission $T(\phi_S)$ of an $N = 7$ gyro for $\alpha = 0$ (blue line), $\alpha = 0.1$ (green line), 0.5 (purple line), 0.9 (red line), and 2.0 dB/cm (teal line). Crosses represent the location of the maximum scale factor. Here, $\kappa = 0.1$ and $R = 300$ μm.

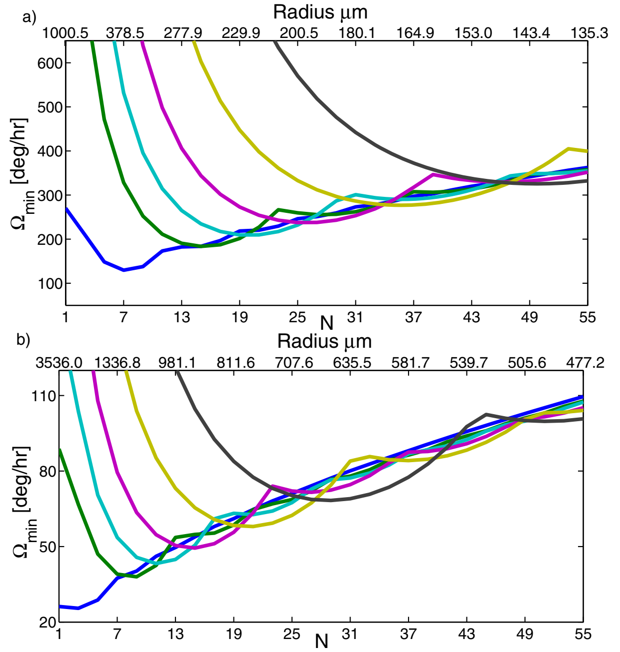

Fig. 5. $\Omega_{\min}$ versus N for $\alpha = 0.06$ dB/cm confined to an area of (a) 4 mm^2 and (b) 50 mm^2 with $\kappa = 0.1$ (blue lines), 0.35 (green lines), 0.5 (teal lines), 0.65 (purple lines), 0.8 (yellow lines), and 0.95 (black lines).

transmission resonance resulting in a transmission slope that is much less sensitive to α.

Finally, we consider the effect of κ on $\Omega_{\min}$ for device areas of 4 mm^2 and 50 mm^2 in Fig. 5. In this case we chose a specific resonator material with very low losses, Hydex, for which $\alpha = 0.06$ dB/cm [3]. The results indicate that the smallest $\Omega_{\min}$ occurs for the smallest value of κ in the range of $0.1 < \kappa < 1$. Furthermore, as κ is increased, the effective length of the CROW decreases, $L_{\mathrm{eff}} = \pi RN/\kappa$, resulting in less attenuation of the signal due to propagation losses, $\sim \exp(-\alpha L_{\mathrm{eff}})$ that causes the minimum of $\Omega_{\min}$ to shift to larger N. However, the minimum value of $\Omega_{\min}$ simultaneously increases with increasing κ due to the reduced number of times the light circulates in each resonator, which acts to magnify the Sagnac effect.

In conclusion, for very small losses $\alpha \lesssim 10^{-3}$ dB/cm the CROW gyro with N resonators has worse sensitivity than a single ring resonator of equivalent circumference for small values of N, while for larger N the CROW yields better sensitivities. By contrast, when the device area is constrained, the CROW has better sensitivity than the single resonator for any value of N and maximizing N by using the smallest possible resonators leads to the best sensitivity. In the presence of resonator losses, it was found that the single resonator sensitivity is less sensitive to increases in α than is the CROW, and for $\alpha \gtrsim 0.1$ dB/cm the single resonator achieves greater sensitivities than any CROW. These results could be readily extended to similar optical gyroscope designs using coupled evanescent waves, such as the fiber microcoil optical gyroscope [15].

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References

1. C. Ciminelli, F. Dell'Olio, C. E. Campanella, and M. N. Armenise, Adv. Opt. Photon. **2**, 370 (2010).
2. V. M. N. Passaro, C. de Tullio, B. Troia, M. La Notte, G. Giannoccaro, and F. De Leonardis, Sensors **12**, 15558 (2012).
3. F. Morichetti, C. Ferrari, A. Canciamilla, and A. Melloni, Laser Photonics Rev. **6**, 74 (2012).
4. J. Scheuer and A. Yariv, Phys. Rev. Lett. **96**, 053901 (2006).
5. M. A. Terrel, M. J. F. Digonnet, and S. Fan, J. Lightwave Technol. **27**, 47 (2009).
6. M. Terrel, M. J. F. Digonnet, and S. Fan, Laser Photonics Rev. **3**, 452 (2009).
7. J. R. E. Toland, Z. A. Kaston, C. Sorrentino, and C. P. Search, Opt. Lett. **36**, 1221 (2011).
8. C. Sorrentino, J. Toland, and C. P. Search, Opt. Express **20**, 354 (2012).
9. B. Z. Steinberg, J. Scheuer, and A. Boag, J. Opt. Soc. Am. B **24**, 1216 (2007).
10. R. Novitski, B. Z. Steinberg, and J. Scheuer, Phys. Rev. A **85**, 023813 (2012).
11. D. Hah and D. Zhang, Opt. Express **18**, 18200 (2010).
12. J. K. S. Poon, J. Scheuer, S. Mookherjea, G. T. Paloczi, Y. Huang, and A. Yariv, Opt. Express **12**, 90 (2004).
13. J. K. S. Poon, J. Scheuer, Y. Xu, and A. Yariv, J. Opt. Soc. Am. B **21**, 1665 (2004).
14. M. A. Guillen-Torres, E. Cretu, N. A. F. Jaeger, and L. Chrostowski, J. Lightwave Technol. **30**, 1802 (2012).
15. J. Scheuer, Opt. Lett. **34**, 1630 (2009).